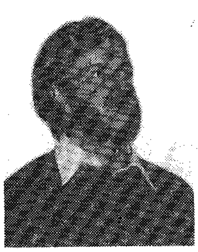

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Analysis and Design of Branch-Line Hybrids with Coupled Lines

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Abstract —**The scattering parameters of four-ports consisting of coupled lines with coupled or uncoupled connecting branches are derived in terms of the even- and odd-mode impedances and the lengths of the lines. These are used to analyze and formulate basic design procedures for the application of these structures as directional couplers including 0° and 90° 3-dB hybrids. The proposed new structures are quite compact at lower frequencies as compared to conventional uncoupled branch-line and rat-race hybrids. The results for the case of the coupled-line four-port with uncoupled branch lines also lead to closed-form expressions for the lengths and impedances of the lines required to nullify the effect of coupling between the main lines of a conventional branch-line coupler for use at higher frequencies. The measured response of the fabricated couplers is in good agreement with the theoretical predictions.**

I. Introduction

BRANCH-LINE HYBRIDS [1] are used extensively at microwave frequencies for a host of applications including phase shifters, balanced amplifiers, and in measurement systems. The major problems arising in the design of such hybrids are the junction effects, coupling between the lines, and dispersion effects. In addition, single-section hybrids are essentially narrow-band structures unless they are designed with matching sections at all the ports [2]. Methods to modify the design to incorporate the junction effects [3]–[6] and various analytical, computational, and experimental (based on heuristic logic) techniques to increase the bandwidth of such structures have also been studied [2], [7]–[9]. However, analytical methods to incorporate the effect of coupling between the lines, particularly the low-impedance lines, which can be significant at higher frequencies, are not available. In this paper, the analysis and basic design procedures for branch-line hybrids with coupled strips as shown in Fig. 1 are presented. Fig. 1(a) represents the conventional branch-line coupler at high frequencies where the physical length of the lines becomes small resulting in significant coupling between parallel lines. The proposed new four-ports of Fig. 1(b) and (c) are relatively compact structures where both the main lines and the branch lines are intentionally coupled. The scattering parameters of these four-ports consisting of coupled lines with coupled or uncoupled branches are derived in terms of even- and odd-mode impedances and the lengths of various lines. These are then used to determine the impedances and lengths of various lines such that the four-port is matched and the division of power between the coupled and the direct ports is in accordance with a desired specified value (e.g., equal for a 3-dB coupler) at the center frequency of the coupler. It is shown that the effect of coupling between the low-impedance lines which may become significant at higher frequencies (e.g., X-band) for conventional branch-line couplers [1] can be compensated by modifying the length and impedances of the lines. In addition, it is shown that compact hybrids consisting of coupled strips with folded branches having frequency response characteristics similar to the conventional branch-

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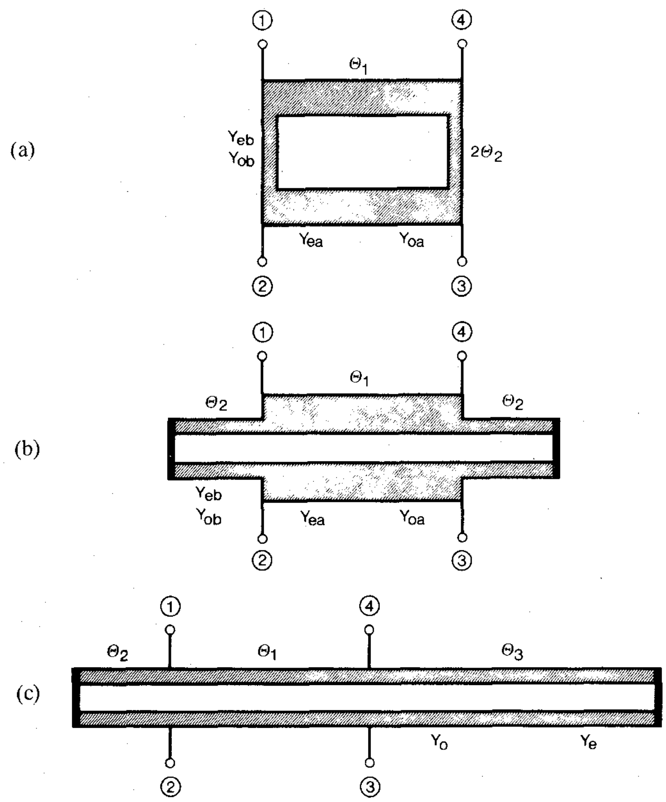


Fig. 1. Schematics of branch-line hybrids with coupled lines. (a) Branch-line coupler. (b) Coupled-line 90° hybrid with folded branch lines. (c) Coupled-line 0° rat-race hybrid.

line and rat-race hybrids can be realized. The results obtained give simple closed-form expressions for the design of such hybrids.

II. The Four-Port Parameters

The four-port parameters of the structures shown in Fig. 1 can be readily found in terms of the equivalent two-ports and their parameters for the even- and odd-modes of excitation. The even- and odd-mode equivalent circuits together with the corresponding admittance parameters for these structures are shown in Table I. These admittances on the general circuit *ABCD* parameters derived similarly can be used to find the immittance on the scattering matrix of the four-port [1], [10]. For example, the scattering parameters of the symmetrical four-ports of Fig. 1(a) and (b) are given by

$$S_{11}=S_{22}=S_{33}=S_{44}=\tfrac{1}{2}[\Gamma_e+\Gamma_o] \quad (1a)$$

$$S_{12}=S_{21}=S_{34}=S_{43}=\tfrac{1}{2}[\Gamma_e-\Gamma_o] \quad (1b)$$

$$S_{14}=S_{41}=S_{23}=S_{32}=\tfrac{1}{2}[T_e+T_o] \quad (1c)$$

$$S_{13}=S_{31}=S_{24}=S_{42}=\tfrac{1}{2}[T_e-T_o] \quad (1d)$$

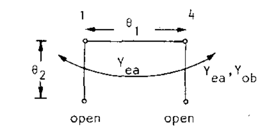
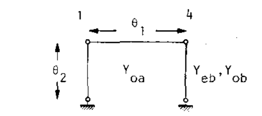
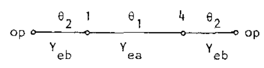
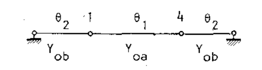
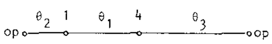
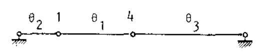
where

$$\Gamma_{e,o}=\frac{1-Y_{11e,o}^2+Y_{14e,o}^2}{(1+Y_{11e,o})^2-Y_{14e,o}^2} \quad (2a)$$

and

$$T_{e,o}=\frac{-2Y_{14e,o}}{(1+Y_{11e,o})^2-Y_{14e,o}^2}. \quad (2b)$$

TABLE I
THE EQUIVALENT TWO-PORT EVEN- AND ODD-MODE ADMITTANCE PARAMETERS

	Even-mode	Odd-mode
Circuit (Fig 1a)		
$Y_{11}=Y_{44}$	$-j\,Y_{ea}\cot\theta_1+j\,\frac{Y_{eb}+Y_{ob}}{2}\tan\theta_2$	$-j\,Y_{oa}\cot\theta_1-j\,\frac{Y_{eb}+Y_{ob}}{2}\cot\theta_2$
$Y_{14}=Y_{41}$	$j\,Y_{ea}\csc\theta_1-j\,\frac{Y_{ob}-Y_{eb}}{2}\tan\theta_2$	$j\,Y_{oa}\csc\theta_1+j\,\frac{Y_{ob}-Y_{eb}}{2}\cot\theta_1$
Circuit (Fig 1b)		
$Y_{11}=Y_{44}$	$-j\,Y_{ea}\cot\theta_1+j\,Y_{eb}\tan\theta_2$	$-j\,Y_{oa}\cot\theta_1-j\,Y_{ob}\cot\theta_2$
$Y_{14}=Y_{41}$	$j\,Y_{ea}\csc\theta_1$	$j\,Y_{oa}\csc\theta_1$
Circuit (Fig 1c)		
Y_{11}	$-j\,Y_e(\cot\theta_1-\tan\theta_2)$	$-j\,Y_o(\cot\theta_1+\cot\theta_2)$
Y_{44}	$-j\,Y_e(\cot\theta_1-\tan\theta_3)$	$-j\,Y_0(\cot\theta_1+\cot\theta_3)$
$Y_{14}=Y_{41}$	$j\,Y_e\csc\theta_1$	$j\,Y_o\csc\theta_1$

Similar expressions are found for the nonsymmetrical structure of Fig. 1(c). The even- and odd-mode admittances are normalized with respect to the terminating admittance Y_o at all the ports. It should be noted that in deriving these expressions, the effects of unequal even- and odd-mode phase velocities and dispersion have been neglected. They can, however, be readily included in all the above expressions.

All the four-ports can be analyzed in terms of their scattering parameters leading to the lengths and impedances of various sections of the structure for possible application as directional couplers including the 3-dB hybrids.

III. Coupler Analysis and Design

For an ideal coupler, all the four-ports must be matched so that

$$S_{ii}=0, \qquad \text{for } i=1, 2, 3, \text{ and } 4$$

that is, either

$$\Gamma_e=\Gamma_o=0 \quad (3a)$$

or

$$\Gamma_e=-\Gamma_o. \quad (3b)$$

It is found that for the couplers of Fig. 1(a) and (b) (directional couplers with a phase difference of $\pi/2$ between the direct and the coupled ports), (3a) must be satisfied, i.e., the network must be matched for both even-

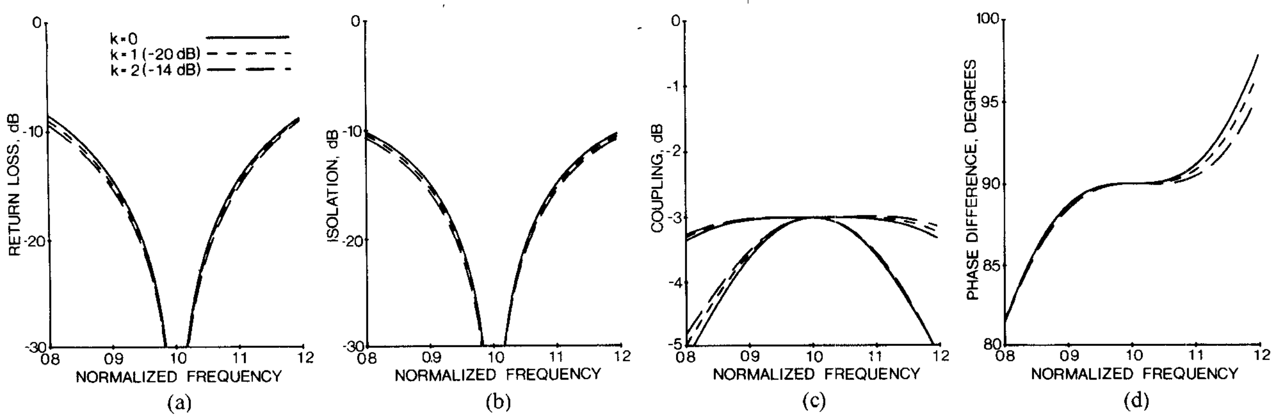


Fig. 2. Frequency response of 3-dB compensated branch-line hybrids for different values of coupling factor k between the low-impedance lines $k=(Z_e-Z_o)/(Z_e+Z_o)$. (a) Input return loss. (b) Isolation. (c) Coupling coefficients for coupled and direct ports. (d) Phase difference between coupled and direct ports.

and odd-modes, whereas for the rat-race-type hybrid of Fig. 1(c), (3b) must be satisfied. In general, for all the hybrids shown in Fig. 1, the design is based on the criterion that all the four-ports are matched and that the division of power between the coupled and the direct port ($|S_{13}|^2/|S_{14}|^2$) is in accordance with the specification at the frequency or in the frequency band of interest. The above matching conditions together with the basic design procedure for the structures shown in Fig. 1 are discussed in the following section.

The matching conditions and the expression for power division when the structure is matched for the symmetrical four-ports of Fig. 1(a) and (b) are given by

$$Y_{11e}^2-Y_{14e}^2=Y_{11o}^2-Y_{14o}^2=1 \tag{4}$$

$$S_{14}/S_{13}=\frac{(Y_{14e}+Y_{14o})+Y_{11e}Y_{14o}+Y_{11o}Y_{14e}}{(Y_{14o}-Y_{14e})+Y_{11e}Y_{14o}-Y_{11o}Y_{14e}} \tag{5}$$

where $Y_{11e,o}$ and $Y_{14e,o}$ are the elements of the admittance matrix of the equivalent two-port as given in Table I. The object now is to solve for the even- and odd-mode admittances of various lines and their lengths such that the above equations are satisfied and $|S_{14}/S_{13}|$ has the desired value, e.g., 1 for the 3-dB hybrids. It is seen that this can be accomplished only at discrete values of frequencies and that there are many possible realizable solutions since there are more variables than the number of equations. Let us examine some useful cases.

A. Branch-Line Hybrid with Coupled Main Lines (Fig. 1(a) with $Y_{eb}=Y_{ob}=Y_2$; $Y_{ea}=Y_e$ and $Y_{oa}=Y_o$)

The matching conditions for this case are given by

$$Y_e^2-Y_2^2\tan^2\Theta_2+2Y_2Y_e\tan\Theta_2\cot\Theta_1=1 \tag{6a}$$

$$Y_o^2-Y_2^2\cot^2\Theta_2-2Y_2Y_0\cot\Theta_2\cot\Theta_1=1. \tag{6b}$$

Then, for a 3-dB hybrid, the power division equation (5) gives

$$Y_2^2-Y_eY_o\cot^2\Theta_1+Y_2\cot\Theta_1(Y_o\tan\Theta_2-Y_e\cot\Theta_2)=1. \tag{6c}$$

The set of equations (6) is undetermined since there are five variables and only three equations and, hence, there are many physically realizable solutions. It is seen that (6) can be satisfied at only one frequency over a frequency band of interest, and the bandwidth of the coupler depends on the selection of the variables.

If we chose Y_2 and Θ_2 to be the same as that for the case of the branch-line coupler with uncoupled lines, i.e., $Y_2=1$ and $\Theta_2=\pi/4$ at the center frequency, then the solution for the impedances and the lengths of the main lines found from (6a) through (6c) is given by

$$\cot\Theta_1=\left[\frac{Y_o-Y_e}{2}\right] \text{ and } Y_eY_o=2. \tag{7}$$

The above equations together with the modifications required due to junction effects [3]–[6] can be used to compensate for any coupling between the low-impedance lines in a branch-line hybrid at higher frequencies. The theoretically computed frequency response of the structures satisfying (7) is shown in Fig. 2 as a function of coupling between the main lines as defined by $k=(Y_o-Y_e)/(Y_o+Y_e)$. It is seen that the response of couplers where main low-impedance lines have been modified to compensate for any coupling effects is similar to that of the uncoupled-line hybrids.

B. Coupled-Line Folded Branch Hybrid (Fig. 1(b))

From the expressions for the equivalent even- and odd-mode admittances as given in Table I, it is readily seen that this case is equivalent to that of the branch-line coupler of Fig. 1(a) with uncoupled branches (analyzed in the previous section) having equivalent branch-line admittance and length as given by

$$Y_{2eq}=\sqrt{Y_{eb}Y_{ob}} \text{ and } \tan\Theta_{2eq}=\sqrt{\frac{Y_{eb}}{Y_{ob}}}\tan\Theta_2. \tag{8}$$

That is, the four-port parameters and all the other properties of the folded branch structure of Fig. 1(b) remain the same if we replace the folded branches having even- and odd-mode admittances Y_{eb} and Y_{ob} and length Θ_2 by

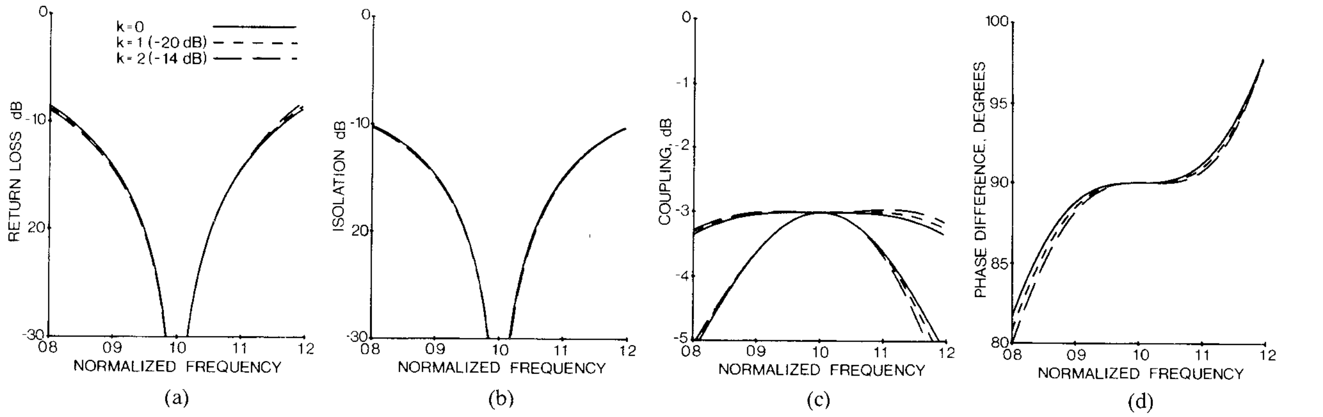


Fig. 3 Frequency response of 3-dB coupled-line 90° hybrids with folded branches for different values of coupling factor k between the main lines. (a) Input return loss. (b) Isolation. (c) Coupling coefficients for coupled and direct ports. (d) Phase difference between coupled and direct ports.

uncoupled branches having admittance Y_{2eq} and length Θ_{2eq}. Again, there are many possible solutions of the matching and power division conditions as given by (6) with Y_2 and Θ_2 replaced by their equivalent values Y_{2eq} and Θ_{2eq}, respectively. For example, a convenient set of parameters satisfying these conditions at the center frequency are given by

$$\begin{aligned} &Y_{eb}Y_{ob}=1\\ &\tan\Theta_2=\sqrt{Y_{ob}/Y_{eb}}\\ &Y_{ea}Y_{oa}=2\\ &\cot\Theta_1=(Y_{oa}-Y_{ea})/2. \end{aligned} \tag{9}$$

The computed frequency response of various structures that satisfy (9) is shown in Fig. 3 as a function of coupling between the branch lines. The response of these hybrids is essentially the same as that of branch-line couplers with uncoupled lines.

The structures are very compact, requiring much smaller area than the ordinary branch-line couplers, particularly at lower frequencies.

C. Rat-Race 3-dB Hybrid (Fig. 1(c))

For this case, if we *a priori* choose the lengths to be the same as that of a hybrid ring, i.e., $\Theta_1=\pi/2$, $\Theta_2=\pi/4$, $\Theta_3=3\pi/4$ at the center frequency, it is found that both the matching and the equal power division conditions cannot be satisfied. The matching condition $S_{ii}=0$, $i=1,2,3,4$ gives $Z_eZ_o=2$ which, together with the 3-dB power division condition, gives $Z_e=Z_o=\sqrt{2}$ which is not possible unless the lines are decoupled. When the lines are coupled ($Z_e\neq Z_o$), it is seen that the 3-dB coupler can be realized either by modifying the length or the impedance of the first section.

For example, choosing Θ_1, such that the structure is matched and the coupling is 3 dB for $\Theta_3=3\,\Theta_2=3\pi/4$ means

$$\Gamma_e=\Gamma_o=0 \tag{10}$$

and

$$\left|\frac{T_e+T_o}{2}\right|=\frac{1}{\sqrt{2}} \tag{11}$$

where

$$\Gamma_{\substack{e\\o}}=\frac{\pm 2\sin\Theta_1+j\sin\Theta_1\left[Z_{\substack{e\\o}}-2/Z_{\substack{e\\o}}\right]}{2\cos\Theta_1+j\sin\Theta_1\left[Z_{\substack{e\\o}}+2/Z_{\substack{e\\o}}\right]}$$

and

$$T_{\substack{e\\o}}=\frac{2}{2\cos\Theta_1+j\sin\Theta_1\left[Z_{\substack{e\\o}}+2/Z_{\substack{e\\o}}\right]}.$$

Again, (10) gives $Z_eZ_o=2$ which, together with (11), leads to

$$\Theta_1=\sin^{-1}\left[(Y_e+Y_o)^2-1\right]^{-1/2}=\tan^{-1}\left(\frac{2}{Z_e-Z_o}\right). \tag{12}$$

The frequency response of typical hybrids satisfying the above conditions at the center frequency is shown in Fig. 4. It is seen that for the cases with small coupling between the lines, the bandwidth of the coupler is improved in terms of the balance between the coupled and the direct port.

IV. Experimental Results

Hybrids consisting of coupled lines with folded coupled branches corresponding to the two cases of Fig. 1(b) and (c) were designed and fabricated in stripline form on Rexolite substrate having a relative dielectric constant $\epsilon_r=2.62$ and height $h=1.59$ mm. The line parameters were chosen to satisfy the matching conditions, and couplers with various coupling factors were designed by utilizing the simple solutions given in the previous section. The coupled-line lengths and even- and odd-mode impedances of a 50-Ω, 2.4-dB folded branch coupler as in Fig. 1(b) were $\Theta_1=83.9°$, $\Theta_2=48.05°$, $Z_{ea}=38.13$ Ω, $Z_{oa}=32.8$ Ω, $Z_{eb}=55.63$ Ω, and $Z_{ob}=44.94$ Ω. The lengths and impedances of a 3-dB, 50-Ω hybrid as in Fig. 1(c) were $\Theta_1=$

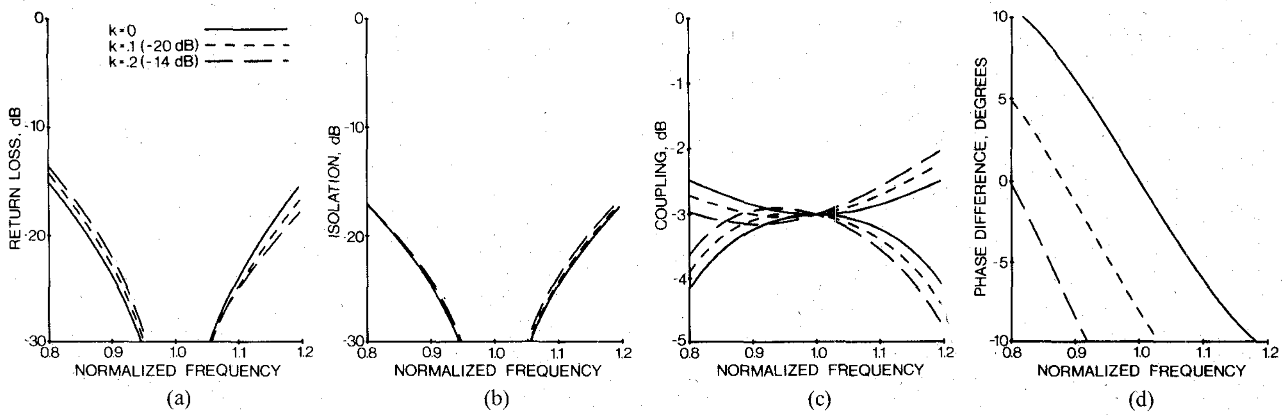


Fig. 4. Frequency response of 3-dB coupled-line 0° rat-race hybrid for different values of coupling between the lines. (a) Return loss. (b) Isolation. (c) Coupling coefficients for coupled and direct ports. (d) Phase difference between coupled and direct ports.

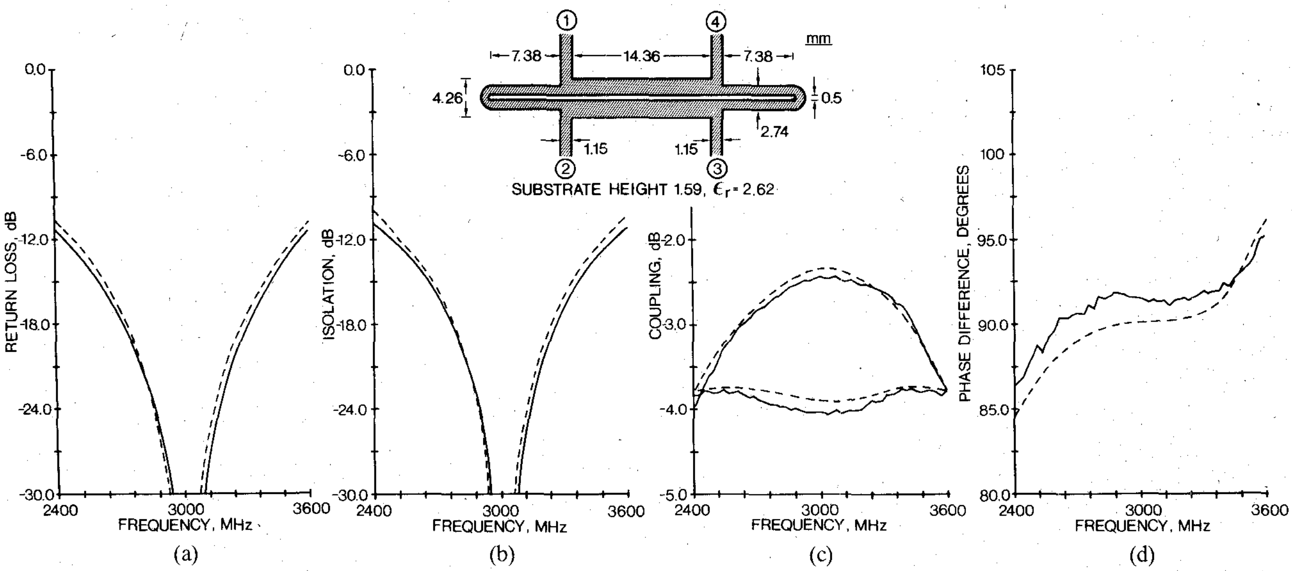


Fig. 5. Frequency response of a folded branch hybrid. —— Experiment. ----- Theory. (a) Return loss. (b) Isolation. (c) Coupling. (d) Phase difference between coupled and direct ports.

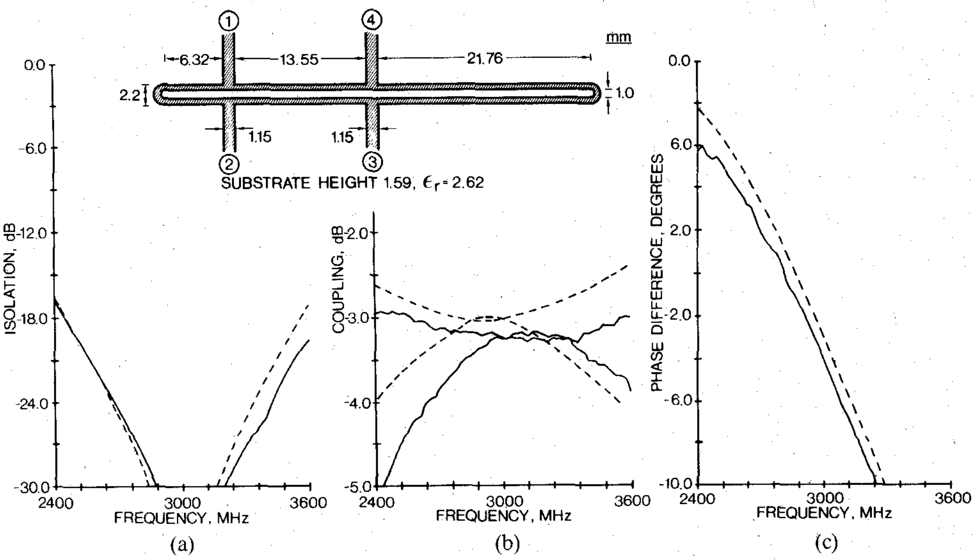


Fig. 6. Frequency response of a 3-dB coupled-line rat-race hybrid. —— Experiment. ----- Theory. (a) Isolation. (b) Coupling. (c) Phase difference between coupled and direct ports.

85.65°, $\Theta_2 = \Theta_3/3 = 45°$, $Z_e = 74.62\ \Omega$, and $Z_o = 67.01\ \Omega$. These parameters translate into the physical strip dimensions given in the insets of Figs. 5 and 6, respectively, for the two cases. The reflection coefficient, isolation, and coupling of these hybrids are shown in Figs. 5 and 6, together with the theoretically computed values. As seen from these figures, the experimental results are in reasonably good agreement with theoretical predictions.

V. Concluding Remarks

Simple expressions for the design of directional couplers consisting of coupled lines with coupled folded branches have been presented. It is shown that the performance of these couplers is similar to the ones with uncoupled branches. However, they do provide for fairly compact structures at lower frequencies, and an added degree of freedom to the designer in the form of coupled lines. The results also lead to the expression for the length of the low-impedance line of a conventional branch-line hybrid such that the effect of any coupling between these lines is compensated for at high frequencies. No attempt was made to optimize the coupler performance in terms of the even- and odd-mode impedances and line lengths of various sections. Such an optimization procedure [9] or the introduction of external and internal matching techniques as in [2] and [8] that have been used for uncoupled branch-line hybrids should also lead to broadbanding of these structures.

Acknowledgment

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